

B.Sc. Part I(H) 2nd Paper Integral Calculus

Reduction Formula (contd)

Q. Establish a reduction formula for  
$$\int \cos^n x \, dx.$$

Soln Let  $I_n = \int \cos^n x \, dx$  — (1)

$$\Rightarrow I_n = \int \cos x \cdot \cos^{n-1} x \, dx$$

Integrating by parts and considering

$\cos^{n-1} x$  as  $u$  and  $\cos x$  as  $v$

$$\Rightarrow I_n = \cos^{n-1} x \int \cos x \, dx - \int \left[ \frac{d}{dx} (\cos^{n-1} x) \int \cos x \, dx \right] dx$$

$$\Rightarrow I_n = \cos^{n-1} x \cdot \sin x - \int (n-1) \cos^{n-2} x \cdot (-\sin x) \cdot \sin x \, dx$$

$$\Rightarrow I_n = \cos^{n-1} x \cdot \sin x + (n-1) \int \cos^{n-2} x \cdot \sin^2 x \, dx$$

$$\Rightarrow I_n = \cos^{n-1} x \cdot \sin x + (n-1) \int \cos^{n-2} x (1 - \cos^2 x) \, dx$$

